$\nabla\cdot(\nabla\times A)=0$

Electromagnetic

Lecture (3)

مركز الفجر
للخدمات العلمية
٠١٠٠٩٢٦٤٠٠٦

Chapter (1)

Part (2)

"Coulomb's Law"

2nd year

communications

مركز الفجر
للخدمات العلمية
٠١٠٠٩٢٦٤٠٠٦

Lecture (3):

Part (2)

Chapter (1)

Coulomb's law and Elec. field

[I] Electric field Intensity:

<< $\bar{E}$: هو مقدار القوة المؤثرة على وحدة الشحنات >>

$$\bar{E} = \frac{\bar{F}}{\text{Charge}}$$

$$\boxed{\bar{E} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{\bar{R}}{|R|^3}}$$

← نستخدم هذا القانون في حالة الـ Point charges.

→ For distributed charges:

line   sur.   vol.

$$d\bar{E} = \frac{dQ}{4\pi\epsilon_0} \cdot \frac{\bar{R}}{|R|^3}$$

$d\bar{E}$: هو شدة المجال الناشئ عن شحنة موزعة على طول صغير dl أو مساحة صغيرة ds أو حجم صغير dv.

$$\therefore dQ = \begin{cases} \rho_l \, dl \longrightarrow \text{line} \\ \rho_s \, dS \longrightarrow \text{Sur.} \\ \rho_v \, dv \longrightarrow \text{Vol.} \end{cases}$$

$$\therefore d\bar{E} = \begin{cases} \dfrac{\rho_l \, dl}{4\pi\epsilon} \cdot \dfrac{\bar{R}}{|R|^3} \rightsquigarrow \text{line} \\ \dfrac{\rho_s \, ds}{4\pi\epsilon} \cdot \dfrac{\bar{R}}{|R|^3} \rightsquigarrow \text{sur.} \\ \dfrac{\rho_v \, dv}{4\pi\epsilon} \cdot \dfrac{\bar{R}}{|R|^3} \longrightarrow \text{vol.} \end{cases}$$

{Summary}

$\Rightarrow$ Point charge: $\bar{E} = \dfrac{Q}{4\pi\epsilon} \cdot \dfrac{\bar{R}}{|R|^3}$

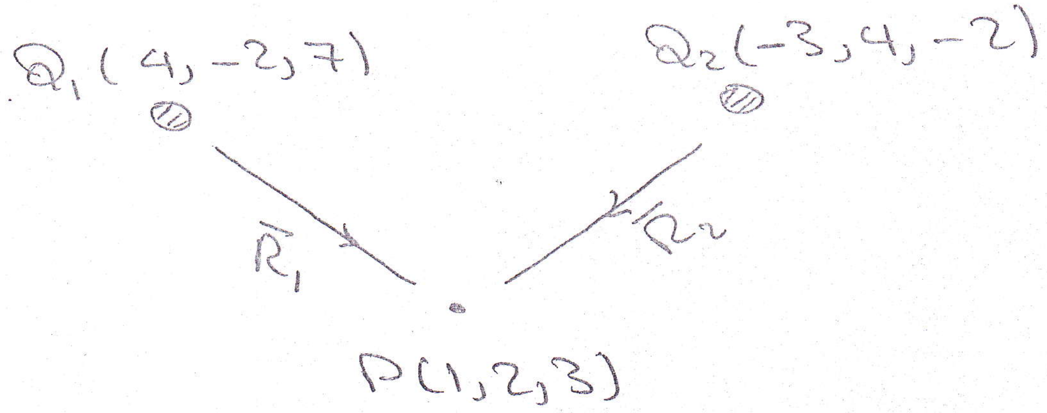
$\Rightarrow$ Line charge: $\bar{E} = \displaystyle\int_l \dfrac{\rho_l \, dl}{4\pi\epsilon} \cdot \dfrac{\bar{R}}{|R|^3}$

$\Rightarrow$ Sur. charge: $\bar{E} = \displaystyle\int_S \dfrac{\rho_s \, dS}{4\pi\epsilon} \cdot \dfrac{\bar{R}}{|R|^3}$

مركز الفجر
لخدمات الطلبة
[illegible]

Sheet (2)

$\boxed{3}$ $Q_1 = 25\,nC$ $\qquad Q_2 = 60\,nC$



(a) find $\bar{E}_p = ??$ "point charges"

$$\bar{R}_1 = (1,2,3) - (4,-2,7) = -3\hat{x} + 4\hat{y} - 4\hat{z}$$

$$\bar{R}_2 = 4\hat{x} - 2\hat{y} + 5\hat{z}$$

$$|R_1| = \sqrt{41} \qquad |R_2| = \sqrt{45}$$

مركز الفجر للخدمات العلمية
٠١٠٠٩٩٣٦٠٠٧

$$\bar{E}_p = \bar{E}_1 + \bar{E}_2$$

$$= \frac{Q_1}{4\pi\epsilon_0} \cdot \frac{\bar{R}_1}{|R_1|^3} + \frac{Q_2}{4\pi\epsilon_0} \cdot \frac{\bar{R}_2}{|R_2|^3}$$

$$= \frac{Q_1}{4\pi\epsilon_0} \cdot \frac{-3\hat{x} + 4\hat{y} - 4\hat{z}}{(41)^{3/2}}$$

$$+ \frac{Q_2}{4\pi\epsilon_0} \cdot \frac{4\hat{x} - 2\hat{y} + 5\hat{z}}{(45)^{3/2}}$$

$$= 4.58\,\hat{x} - 0.15\,\hat{y} + 5.51\,\hat{z} \quad (V/m)$$

(b) find $P(0,y,0)$ at which $E_x = 0$

$$\bar{R}_1 = (0,y,0)-(4,-2,7) = -4\hat{x}+(y+2)\hat{y}-7\hat{z}$$

$$\bar{R}_2 = (0,y,0)-(-3,4,-2) = 3\hat{x}+(y-4)\hat{y}+2\hat{z}$$

$$|R_1| = \sqrt{65+(y+2)^2}$$

$$|R_2| = \sqrt{13+(y-4)^2}$$

$$\bar{E}_p = \frac{Q_1}{4\pi\epsilon}\cdot\frac{\boxed{-4\hat{x}}+(y+2)\hat{y}-7\hat{z}}{[65+(y+2)^2]^{3/2}} + \frac{Q_2}{4\pi\epsilon}\cdot\frac{\boxed{3\hat{x}}+(y-4)\hat{y}+2\hat{z}}{[13+(y-4)^2]^{3/2}}$$

$\Rightarrow$ $E_x = 0$
تركيبة ال E_p في
اتجاه ال x
= الصفر

$$\therefore \frac{Q_1}{\cancel{4\pi\epsilon}}\cdot\frac{-4}{[65+(y+2)^2]^{3/2}} + \frac{Q_2}{\cancel{4\pi\epsilon}}\cdot\frac{3}{[13+(y-4)^2]^{3/2}} = 0$$

$$\therefore 4\underset{100}{\cancel{Q_1}}[13+(y-4)^2]^{3/2} = 3\underset{180}{\cancel{Q_2}}[65+(y+2)^2]^{3/2}$$

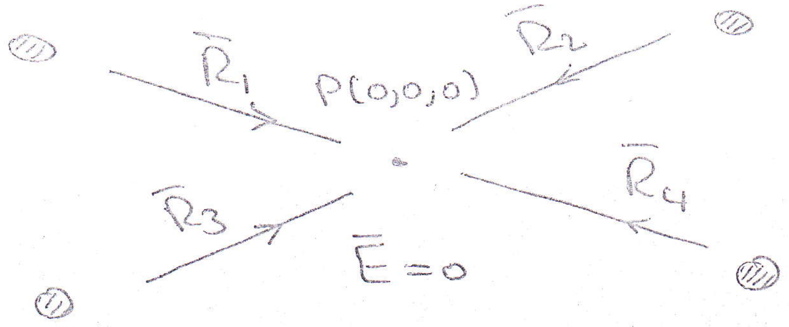
عن طريق حل هذه المعادلة باستخدام الحاسبة :

$$y = -6.886$$

$\therefore$ The point is: $(0, -6.886, 0)$

$\boxed{12}$

$Q_1 = 5nC\ (0,5,0)$ $\qquad$ $Q_2 = -10nC\ (0,-5,0)$



$Q_3 = 15nC\ (-5,0,0)$ $\qquad$ $Q_4 = 20nC\ (x,y,0)$

x, y → are required.

$$\left.\begin{aligned} \bar{R}_1 &= -5\hat{y} \\ \bar{R}_2 &= 5\hat{y} \\ \bar{R}_3 &= 5\hat{x} \end{aligned}\right\} |R_1| = |R_2| = |R_3| = 5$$

$$\bar{R}_4 = -x\hat{x} - y\hat{y} \rightarrow |R_4| = \sqrt{x^2+y^2}$$

$$\bar{E}_P = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \bar{E}_4 = 0$$

$$\frac{Q_1}{4\pi\epsilon} \cdot \frac{-5\hat{y}}{5^3} + \frac{Q_2}{4\pi\epsilon} \cdot \frac{5\hat{y}}{5^3} + \frac{Q_3}{4\pi\epsilon} \cdot \frac{5\hat{x}}{5^3}$$

$$+ \frac{Q_4}{4\pi\epsilon} \cdot \frac{-x\hat{x} - y\hat{y}}{(x^2+y^2)^{3/2}} = 0$$

⇒ x-Comp :

$$\frac{15}{\cancel{4\pi\epsilon}} \cdot \frac{1}{25} - \frac{20x}{\cancel{4\pi\epsilon}\,(x^2+y^2)^{3/2}} = 0$$

$$\therefore \frac{x}{(x^2+y^2)^{3/2}} = 0.03 \longrightarrow (1)$$

$\Rightarrow$ y_Comp :

$$-\frac{5}{\cancel{4\pi\epsilon}}\cdot\frac{1}{25}+\frac{-10}{\cancel{4\pi\epsilon}}\cdot\frac{1}{25}-\frac{20y}{\cancel{4\pi\epsilon}(x^2+y^2)^{3/2}}=0$$

$$\therefore \quad \frac{y}{(x^2+y^2)^{3/2}}=-0.03 \longrightarrow (2)$$

divide (1) ÷ (2) : $\frac{x}{y}=-1 \quad \therefore x=-y \longrightarrow (3)$

from (3) in (1) :

$$\frac{-y}{(2y^2)^{3/2}}=0.03$$

using Calculator : $y=-3.43$

$\therefore x=-y=3.43$

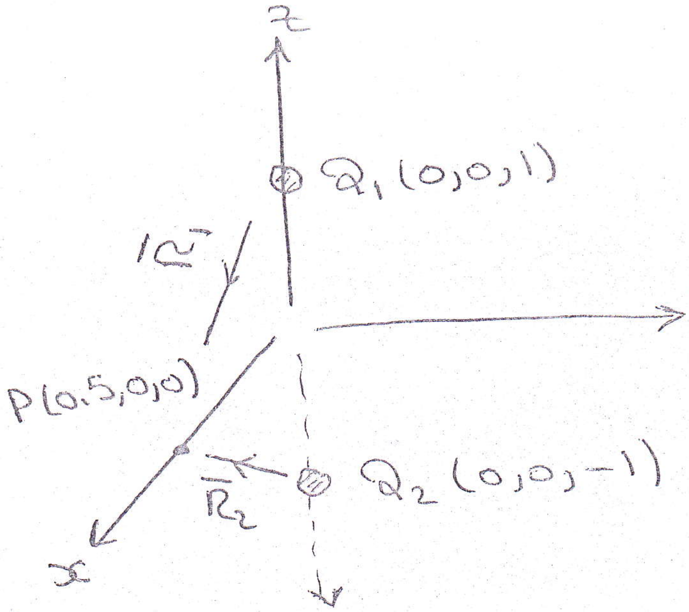
$\therefore$ The point of

Q_4 is $(3.43, -3.43, 0)$.

Sheet (2)

$\boxed{4}$ $Q = 120\ nC$

(a) find $\bar{E}_p\Big|_{(0.5,0,0)}$



$$\bar{R}_1 = 0.5\,\hat{x} - \hat{z} \qquad |R_1| = \frac{\sqrt{5}}{2}$$

$$\bar{R}_2 = 0.5\,\hat{x} + \hat{z} \qquad |R_2| = \frac{\sqrt{5}}{2}$$

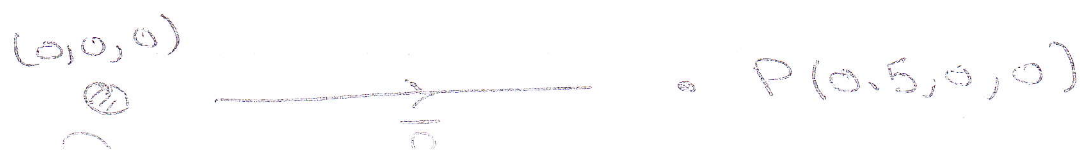
$$\bar{E}_p = \bar{E}_1 + \bar{E}_2$$

$$= \frac{Q}{4\pi\epsilon} \cdot \frac{\bar{R}_1}{|R_1|^3} + \frac{Q}{4\pi\epsilon} \cdot \frac{\bar{R}_2}{|R_2|^3}$$

$$= \frac{Q}{4\pi\epsilon}\left[\frac{0.5\hat{x} - \cancel{\hat{z}}}{\left(\frac{5}{4}\right)^{3/2}} + \frac{0.5\hat{x} + \cancel{\hat{z}}}{\left(\frac{5}{4}\right)^{3/2}}\right]$$

$$= \underline{771.72\,\hat{x}}\ V/m \qquad \#$$

(b) لو اريد استبدال الشحنتين بشحنة عند ال origin بحيث تقوم بتوليد نفس قيمة المجال عند النقطة P.



$\therefore \bar{R} = 0.5\hat{x}$ $\quad |R| = 0.5$

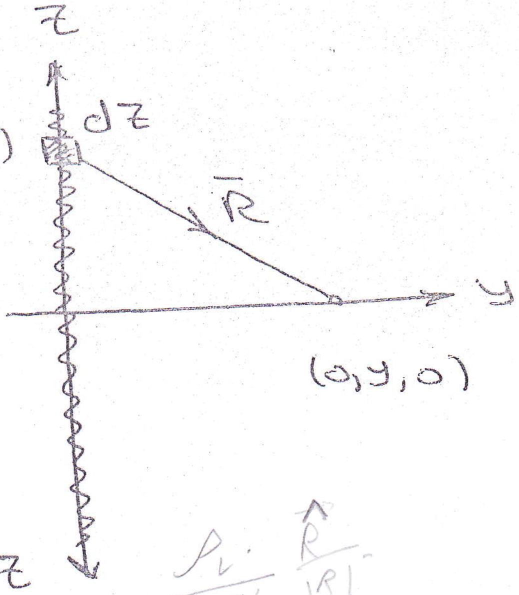
$$\bar{E}_P = 771.72\,\hat{x} = \frac{Q}{4\pi\epsilon} \cdot \frac{0.5\,\hat{x}}{0.5^3}$$

$$\therefore \underline{Q = 21.47\ \text{nC}}$$

Ex Find $\bar{E}$ at Point $P(0,y,0)$ due to an infinite line Charged with Constant ρ_L and Placed on z-axis.

Solution

$$\bar{E}_l = \int_l \frac{\rho_L \, dl}{4\pi\epsilon} \cdot \frac{\bar{R}}{|R|^3}$$



- $dl = dz$
- $\bar{R} = y\hat{y} - z\hat{z}$
- $|R| = \sqrt{y^2 + z^2}$

$$\therefore \bar{E}_l = \frac{\rho_L}{4\pi\epsilon} \int_{-\infty}^{\infty} \frac{y\hat{y} - \cancelto{0}{z\hat{z}}}{(y^2+z^2)^{3/2}} \, dz$$

$\frac{\rho_L}{2\pi\epsilon} \cdot \frac{\hat{R}}{|R|}$

$\frac{\rho_L}{2\pi\epsilon}$

• يمكن ملاحظة أن مركبة المجال فى إتجاه الـ $\hat{z}$ = صفر
وذلك لأن تأثير نصف الخط السفلي يلغي تأثير نصف الخط
العلوى بالنسبة للمركبة فى إتجاه الـ $\hat{z}$.

$$\therefore \bar{E} = \frac{\rho_L y}{4\pi\epsilon} \int_{-\infty}^{\infty} \frac{dz}{(y^2+z^2)^{3/2}} \hat{y}$$

y: ثابت

let $z = y \tan\theta$

$dz = y \sec^2\theta \, d\theta$

$z = -\infty \longrightarrow \theta = -\pi/2$

$$\therefore \bar{E} = \frac{\rho_L y}{4\pi\epsilon} \int_{-\pi/2}^{\pi/2} \frac{y \sec^2\theta \, d\theta}{(y^2 + y^2 \tan^2\theta)^{3/2}} \hat{y}$$

$\rightarrow$ المقام $= (y^2 + y^2 \tan^2\theta)^{3/2} = (y^2)^{3/2} \, (1 + \tan^2\theta)^{3/2}$

$$= y^3 \, (\sec^2\theta)^{3/2} = y^3 . \sec^3\theta$$

$$\therefore \bar{E} = \frac{\rho_L \cancel{y}}{4\pi\epsilon} \int_{-\pi/2}^{\pi/2} \frac{\cancel{y} \, \cancel{\sec^2\theta} \, d\theta}{y^{\cancel{3}} . \sec^{\cancel{3}}\theta} \hat{y}$$

$$= \frac{\rho_L}{4\pi\epsilon y} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \; \hat{y}$$

$$= \frac{\rho_L}{4\pi\epsilon y} . \sin\theta \Big|_{-\pi/2}^{\pi/2} \; \hat{y}$$

$$= \frac{\rho_L}{4\pi\epsilon y} . (1 + 1) \hat{y} = \frac{\rho_L}{2\pi\epsilon y} \hat{y}$$

$$\therefore \boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon} . \frac{\hat{y}}{y}}$$

نستخلص من هذا المثال قانون هام:

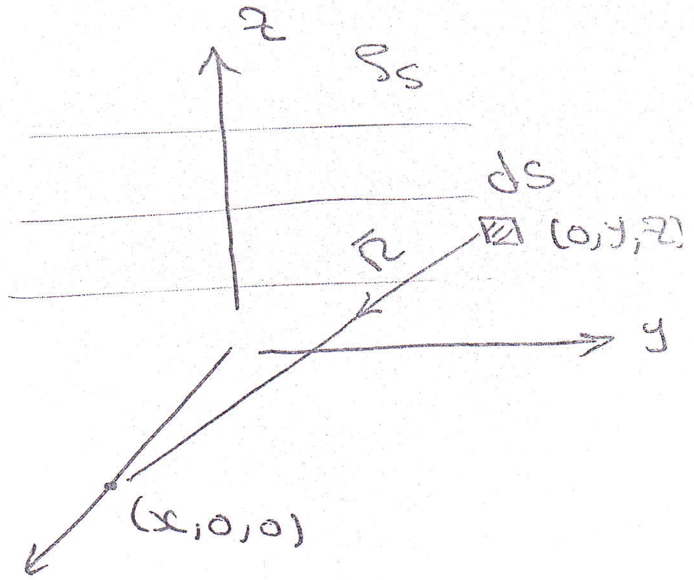
• لأي خط لانهائي الامتداد ومشحون بشحنة (ρ_L) ثابتة:

$\bar{E}_l = \frac{\rho_L}{2\pi\epsilon} . \frac{\hat{l}}{|l|}$ $\qquad |l| :$ البعد العمودي بين الخط والنقطة التي نحسب عندها

$\frac{\rho_L}{\;} . \frac{\bar{l}}{\;}$ $\qquad \left(\hat{l} = \frac{\bar{l}}{\;} \right)$

(Ex) Find $\bar{E}$ at point $P(x,0,0)$ due to an infinite surface charged with const. ρ_s and placed on y_z plane

Solution;



$$\bar{E}_s = \int_s \frac{\rho_s\, ds}{4\pi\epsilon} \cdot \frac{\bar{R}}{|R|^3}$$

$$\bar{R} = x\hat{x} - y\hat{y} - z\hat{z}$$

$$|R| = \sqrt{x^2+y^2+z^2}$$

$$ds = dy\, dz$$

$$\bar{E}_s = \frac{\rho_s}{4\pi\epsilon} \iint_{-\infty}^{\infty} \frac{x\hat{x} \cancelto{0}{- y\hat{y}} \cancelto{0}{- z\hat{z}}}{(x^2+y^2+z^2)^{3/2}}\, dy\, dz$$

يمكن ملاحظة أن تأثير نصف المستوى الأيمن يلغي تأثير نصف المستو
الأيسر في اتجاه $\hat{y}$ وأيضاً تأثير نصف المستوى العلوي يلغي تأثير
نصف المستوى السفلي في اتجاه الـ $\hat{z}$ "وتبقى مركبة المجال في اتجاه
الـ $\hat{x}$ ".

$$\therefore \bar{E}_s = \frac{\rho_s x}{4\pi\epsilon} \underbrace{\iint_{-\infty}^{\infty} \overbrace{\frac{dy}{(x^2+y^2+z^2)^{3/2}}}^{I_1} dz}_{I_2}\, \hat{x}$$

⇛ we have double integration:

⊛ $I_1 = \int_{-\infty}^{\infty} \frac{dy}{(x^2+y^2+z^2)^{3/2}}$ $x , z \rightarrow$ ثوابت

∴ let $y = \sqrt{x^2+z^2}\tan\theta$

$dy = \sqrt{x^2+z^2}\sec^2\theta \, d\theta$

$y = -\infty \rightarrow \theta = -\pi/2$

$y = \infty \rightarrow \theta = \pi/2$

$$\therefore I_1 = \int_{-\pi/2}^{\pi/2} \frac{\sqrt{x^2+z^2}\sec^2\theta \, d\theta}{(x^2+z^2+(x^2+z^2)\tan^2\theta)^{3/2}}$$

→ المقام $= (x^2+z^2)^{3/2}\left[1+\tan^2\theta\right]^{3/2}$

$= (x^2+z^2)^{3/2}(\sec^2\theta)^{3/2}$

$= (x^2+z^2)^{3/2}\sec^3\theta$

$$\therefore I_1 = \frac{1}{x^2+z^2}\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= \frac{1}{x^2+z^2}\sin\theta\Big|_{-\pi/2}^{\pi/2} = \frac{2}{x^2+z^2}$$

⊛ $I_2 = \int_{-\infty}^{\infty} I_1 \, dz = \int_{-\infty}^{\infty} \frac{2}{x^2+z^2} dz$

$$\therefore I_2 = \frac{2}{x^2}\int_{-\infty}^{\infty} \frac{dz}{1+\left(\frac{z}{x}\right)^2}$$

$$= \frac{2}{x^2}\tan^{-1}\left(\frac{z}{x}\right)\cdot x \Big|_{-\infty}^{\infty}$$

$$= \frac{2}{x}\left\{\frac{\pi}{2}+\frac{\pi}{2}\right\} = \frac{2\pi}{x}$$

$\tan^{-1} z$

تفاضل $\Downarrow$

$\frac{1}{1+z^2}$

$$\therefore \bar{E} = \frac{\rho_s \cancel{x}}{\cancel{4\pi}\,\epsilon}\cdot\frac{\cancel{2\pi}}{\cancel{x}}\,\hat{x}$$

2

$$= \boxed{\frac{\rho_s}{2\epsilon}\,\hat{x}}$$

$\frac{\rho_s}{2\epsilon_0}\,\frac{x\,\hat{x}}{x^2}$

$\frac{\rho_s}{2\epsilon_0 x^2}\,\hat{x}$

• نستخلص من هذا المثال أيضاً قانون هام :

لأى سطح لانهائى الإمتداد مشحون بشحنة ρ_s ثابتة .

$$\bar{E}_s = \frac{\rho_s}{2\epsilon}\,\hat{n}$$

حيث

متجه الوحدة العمودى

على السطح وفى اتجاه

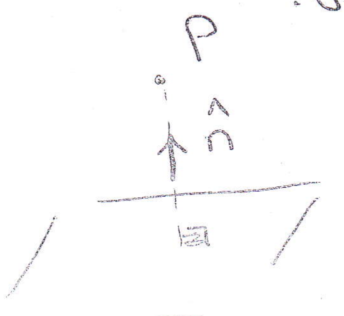
النقطة التى نحسب عندها

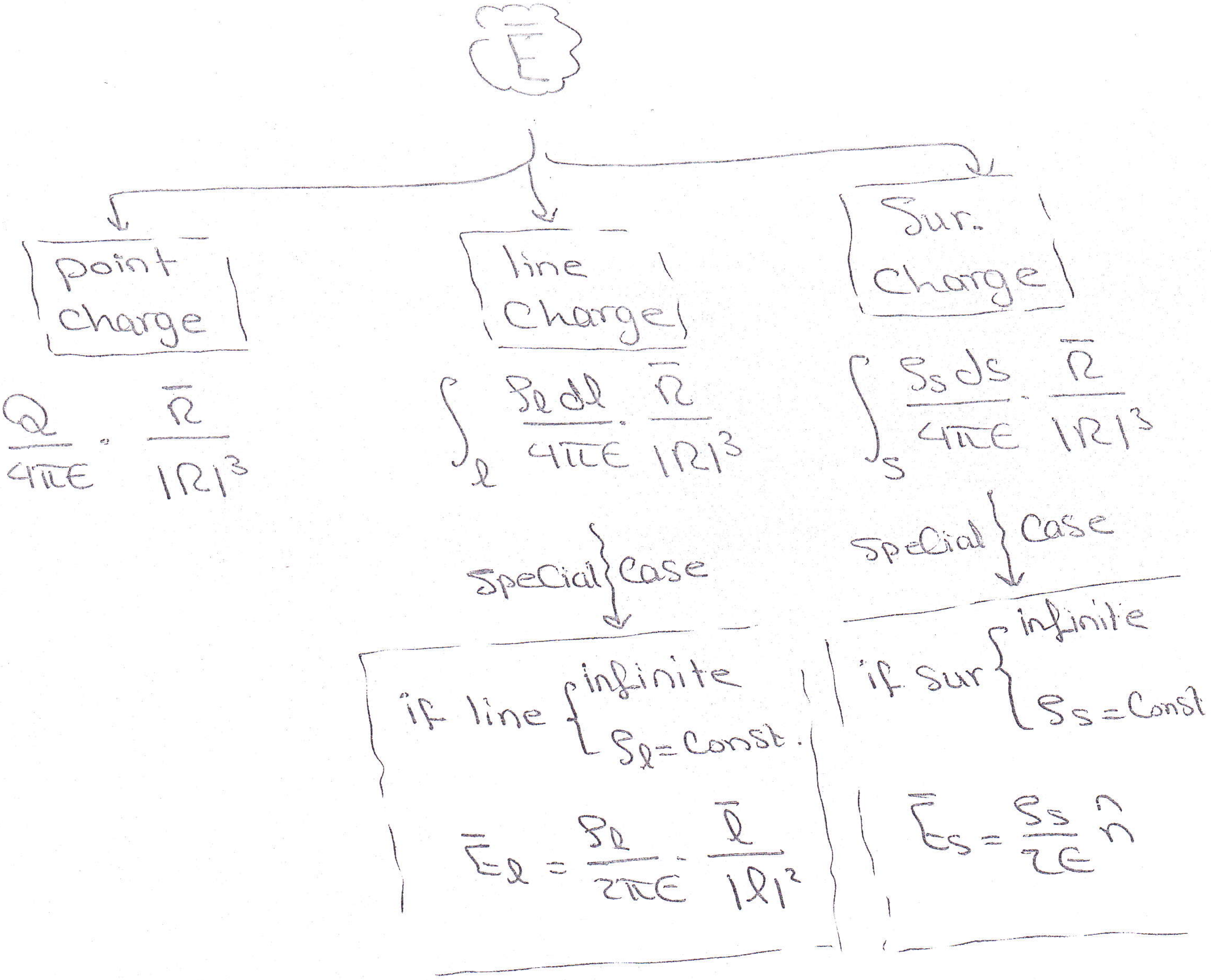
المجال .

$\therefore$ if sur $\begin{cases}\text{infinite}\\ \rho_s = \text{Const}\end{cases}$

$$\therefore \boxed{\bar{E}_s = \frac{\rho_s}{2\epsilon}\,\hat{n}}$$

حفظ




$\bar{E}$
point charge
line Charge
Sur. Charge
Special Case
Special Case
if line $\begin{cases} \text{infinite} \\ \rho_l = \text{Const.} \end{cases}$
if sur $\begin{cases} \text{infinite} \\ \rho_s = \text{Const} \end{cases}$

$$\frac{Q}{4\pi\epsilon}\cdot\frac{\bar{R}}{|R|^3}$$

$$\int_l \frac{\rho_l\, dl}{4\pi\epsilon}\cdot\frac{\bar{R}}{|R|^3}$$

$$\int_s \frac{\rho_s\, ds}{4\pi\epsilon}\cdot\frac{\bar{R}}{|R|^3}$$

$$\bar{E}_l = \frac{\rho_l}{2\pi\epsilon}\cdot\frac{\bar{l}}{|l|^2}$$

$$\bar{E}_s = \frac{\rho_s}{2\epsilon}\hat{n}$$

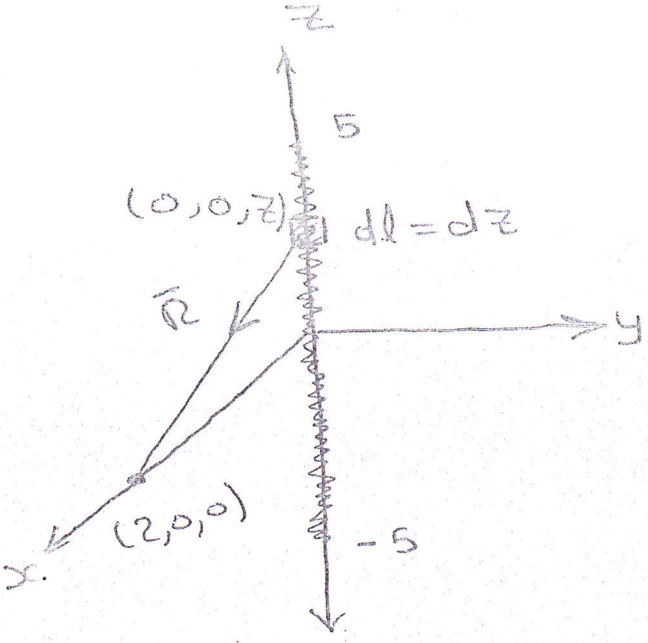
$$\int_l \frac{\rho_l\, dl}{4\pi\epsilon}\frac{\bar{R}}{|R|^3}$$

$$\downarrow$$

$$\frac{\rho_l}{2\pi\epsilon_0}\frac{\bar{l}}{|l|^2} \qquad \frac{\rho_s}{2\epsilon}\hat{n}$$

5) $\rho_L = 20\ \text{nC/m}$

$\bar{E}$ at $(2,0,0) = ?$



Solution

$$\bar{E} = \int \frac{\rho_L\, dl}{4\pi\epsilon_0} \cdot \frac{\bar{R}}{|R|^3}$$

$dl = dz$ (on z-axis).

$\bar{R} = (2,0,0) - (0,0,z) = 2\hat{x} - z\hat{z}$

$|R| = \sqrt{4+z^2}$

$$\therefore \bar{E} = \int_{-5}^{5} \frac{\rho_L\, dz}{4\pi\epsilon_0} \cdot \frac{2\hat{x} - \cancelto{0}{z\hat{z}}}{(4+z^2)^{3/2}}$$

- يمكن ملاحظة أن مركبة المجال في إتجاه ال $\hat{z}$ تساوي صفر لأن تأثير الجزء السفلي من الخط المشحون يلغي تأثير الجزء العلوي في إتجاه ال $\hat{z}$.

$$\bar{E} = \frac{2\rho_L}{4\pi\epsilon_0} \underbrace{\int_{-5}^{5} \frac{dz}{(4+z^2)^{3/2}}} \hat{x}$$

يمكن حله باستخدام الآلة
الحاسبة ويمكن حله عن طريق
التعويض

باستخدام التعويض :

$$I = \int_{-5}^{5} \frac{dz}{(4+z^2)^{3/2}}$$

let $z = 2\tan\theta$

$dz = 2\sec^2\theta \, d\theta$

تعويض محفوظ
(هام)

$z = -5 \rightarrow \theta = \tan^{-1}\left(\frac{-5}{2}\right) = \theta_1$

$z = 5 \rightarrow \theta = \tan^{-1}\left(\frac{5}{2}\right) = \theta_2$

$$\therefore I = \int_{\theta_1}^{\theta_2} \frac{2\sec^2\theta \, d\theta}{(4+4\tan^2\theta)^{3/2}}$$

$$= \frac{2}{4^{3/2}} \int_{\theta_1}^{\theta_2} \frac{\sec^2\theta}{(1+\tan^2\theta)^{3/2}} \, d\theta$$

$$\rightarrow (1+\tan^2\theta)^{3/2} = (\sec^2\theta)^{3/2} = \sec^3\theta$$

$$\therefore I = \frac{1}{4} \int_{\theta_1}^{\theta_2} \frac{\sec^2\theta}{\sec^3\theta} \, d\theta$$

$$= \frac{1}{4} \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta$$

$$= \frac{1}{4} \sin\theta \Big|_{\theta_1}^{\theta_2}$$

$$= \frac{1}{4} \left[\sin\theta_2 - \sin\theta_1 \right]$$

$$= 0.464$$

نفس القيمة الذي يمكن حسابها من الآلة الحاسبة .

Mansoura University
Faculty of Engineering
Communication Engineering Department

Second Year Communication
Electromagnetic Fields
Sheet (2)

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[1] Point charges of 50nC each are located at A (1, 0, 0), B (-1, 0, 0), C (0, 1,0), and D (0,-1, 0) in free space. Find the total force on the charge at A.

[2] Three point charges are positioned in the x-y plane as follows: 5nC at y = 5 cm, −10nC at y = −5 cm, and 15nC at x = −5 cm. Find the required x-y coordinates of a 20nC fourth charge that will produce a zero electric field at the origin.

[3] Let a point charge Q_1 = 25nC be located at P_1 (4, −2, 7) and a charge Q_2 = 60nC be at P_2 (−3, 4, −2). (a) Find E at P_3 (1, 2, 3). (b) At what point on the y axis is $E_x = 0$?

[4] Point charges of 120nC are located at A(0, 0, 1) and B(0, 0,-1) in free space.(a) Find **E** at P (0.5, 0, 0).(b) What single charge at the origin would provide the identical field strength?

[5] As shown in Fig.1 Charge is distributed along the z axis between z = ± 5 m with a uniform density ρ_l = 20nC/m. Determine **E** at (2, 0, 0).

[6] A uniform line charge of 16nC/m is located along the line defined by y = −2, z = 5.

(a) Find **E** at P (1, 2, 3).

(b) Find **E** at that point in the z = 0 plane where the direction of **E** is given by (1/3) a_y − (2/3) a_z.

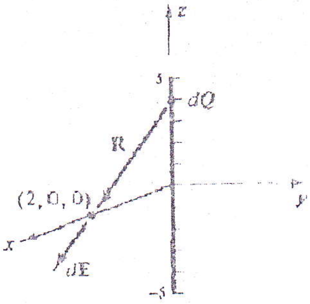

Fig.1

[7] An infinite line charge of charge density ρ_l = 2nC/m lies along the x axis in free space, while two point charges of 8nC are located at (0, 0, 1) and (0, 0,-1). Find E at (2, 3, -4). To what value should ρ_l be changed to cause E to be zero at (0, 0, 3)?

[8] A uniform line charge of ρ_l = 2πnC/m lies along the y axis, while uniform surface charge densities of + 0.1 and - 0.1nC/m^2 exist on the planes z = 3 m and z = - 4 m, respectively. Find E at the point P (1,-7, 2).

[9] Find the stream line equation of the field $\mathbf{E} = \rho^2 a_\rho - \sec\varphi \, a_\varphi$.